# OSCILLATIONS

## FACT/DEFINITION TYPE QUESTIONS

**1.** A periodic to-and-fro motion is called
(a) Periodic motion (b) orbital motion
(c) oscillatory motion (d) rectilinear motion

**2.** A particle moves in a circular path with a uniform speed. Its motion is
(a) periodic (b) oscillatory
(c) simple harmonic (d) angular simple harmonic

**3.** Force acting in simple harmonic motion is always
(a) directly proportional to the displacement
(b) inversely proportional to the displacement
(c) directly proportional to the square of displacement
(d) inversely proportional to the square of displacement

**4.** For which of the following conditions will there be an effect on the periodic-time of the swing, if a girl is sitting on the swing ?
(a) Another girl sits by her side
(b) The girl sitting on the swing stands up
(c) Both (a) and (b)
(d) None of these

**5.** A simple harmonic motion is represented by $y(t) = 10 \sin(20t + 0.5)$. The frequency is
(a) $\frac{20}{2\pi}$ Hz (b) $\frac{10}{2\pi}$ Hz
(c) $\frac{\pi}{2\pi}$ Hz (d) $2\pi$ Hz

**6.** Which of the following expressions is that of a simple harmonic progressive wave ?
(a) $a \sin \omega t$ (b) $a \sin(\omega t)\, cox\, kx$
(c) $a \sin(\omega t - kx)$ (d) $a \cos kx$

**7.** Oscillation and vibration both are ...X... .
Here, $X$ refers to
(a) different concepts (b) same concepts
(c) straight line motions (d) None of these

**8.** Which of the following is a simple harmonic motion?
(a) Particle moving through a string fixed at both ends.
(b) Wave moving through a string fixed at both ends.
(c) Earth spinning about its axis.
(d) Ball bouncing between two rigid vertical walls.

**9.** The displacement of a particle is given by $\vec{r} = A(\vec{i} \cos \omega t + \vec{j} \sin \omega t)$. The motion of the particle is
(a) simple harmonic
(b) on a straight line
(c) on a circle
(d) with constant acceleration.

**10.** The displacement of a particle in simple harmonic motion in one time period is
(a) A (b) 2A
(c) 4A (d) Zero

**11.** In a periodic motion, displacement is a periodic function of time given by
(a) $f(t) = A \cos \omega t$
(b) $f(t) = \sin \omega t$
(c) $f(t) = A \sin \omega t + A \cos \omega t$
(d) All of these

**12.** Which of the following is a non-periodic motion as represented by the equation? ($\omega$ is any positive constant and $t$ is time)
(a) $A \sin(\omega t + \phi)$ (b) $A \cos(\omega t + \phi)$
(c) $e^{-2\omega t}$ (d) All of these

**13.** The function $\sin^2(\omega t)$ represents
(a) a periodic, not simple harmonic motion with a period $\frac{\pi}{\omega}$
(b) a periodic, not simple harmonic motion with a period $\frac{2\pi}{\omega}$
(c) a simple harmonic motion with a period $\frac{\pi}{\omega}$
(d) a simple harmonic motion with a period $\frac{2\pi}{\omega}$

**14.** A child swinging on swing in sitting position stands up. The time period of the swing will
(a) increase
(b) decrease
(c) remain same
(d) increase if the child is tall and decrease if the child is short.

**15.** A particle moves on the X-axis according to the equation $x = A + B \sin \omega t$. The motion is simple harmonic with amplitude
(a) A (b) B
(c) A+B (d) $\sqrt{A^2 + B^2}$

16. The physical quantity which remains constant in simple harmonic motion is
(a) kinetic energy (b) potential energy
(c) restoring force (d) frequency

17. In simple harmonic motion force is directly proportional to ...X... of the body from the mean position. Here, *A* refers to
(a) path length
(b) displacement
(c) square of displacement
(d) None of the above

18. If in a motion $\mathbf{F} \propto \boldsymbol{x}$ it is known as ...*X*... motion. Here, *X* refers, to (with negative constant of proportionality)
(a) linear harmonic (b) non-linear harmonic
(c) cubic harmonic (d) projectile

19. Suppose a tunnel is dug along a diameter of the earth. A particle is dropped from a point, a distance h directly above the tunnel, the motion of the particle is
(a) simple harmonic (b) parabolic
(c) oscillatory (d) non-periodic

20. A particle moves in a circular path with a continuously increasing speed. Its motion is
(a) periodic (b) oscillatory
(c) simple harmonic (d) None of these

21. The motion of a particle is given by $x = A\sin\omega t + B\cos\omega t$. The motion of the particle is
(a) not simple harmonic
(b) simple harmonic with amplitude (A–B)/2
(c) simple harmonic with amplitude (A + B)/2
(d) simple harmonic with amplitude $\sqrt{A^2+B^2}$

22. A system exhibiting SHM must possess
(a) inertia only
(b) elasticity as well as inertia
(c) elasticity, inertia and an external force
(d) elasticity only

23. For a body executing simple harmonic motion, which parameter comes out to be non-periodic ?
(a) Displacement (b) Velocity
(c) Acceleration (d) None of these

24. In SHM, the acceleration is directly proportional to :
(a) time (b) linear velocity
(c) displacement (d) frequency

25. Velocity of a body moving in SHM is
(a) $\omega\sqrt{a^2+y^2}$ (b) $\omega^2\sqrt{a^2+y^2}$
(c) $\omega\sqrt{a^2-y^2}$ (d) $\omega^2\sqrt{a^2-y^2}$

26. The average acceleration in one time period in a simple harmonic motion is
(a) $A\omega^2$ (b) $A\omega^2/2$
(c) $A\omega^2/\sqrt{2}$ (d) zero

27. The graph plotted between the velocity and displacement from mean position of a particle executing SHM is
(a) circle (b) ellipse
(c) parabola (d) straight line

28. Acceleration of a particle executing SHM, at it's mean position is
(a) infinity (b) variable
(c) maximum (d) zero

29. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is
(a) $\pi$ (b) $0.707\,\pi$
(c) zero (d) $0.5\,\pi$

30. Which one of the following equations of motion represents simple harmonic motion?
(a) Acceleration $= -k(x+a)$
(b) Acceleration $= k(x+a)$
(c) Acceleration $= kx$
(d) Acceleration $= -k_0x + k_1x^2$
where k, $k_0$, $k_1$ and a are all postive.

31. Which of the following quantities are always negative in a simple harmonic motion ?
(a) $\vec{F}.\vec{r}$. (b) $\vec{v}.\vec{r}$.
(c) $\vec{a}.\vec{r}$. (d) Both (a) & (c)

32. Which of the following is true about total mechanical energy of SHM ?
(a) It is zero at mean position.
(b) It is zero at extreme position.
(c) It is always zero.
(d) It is never zero.

33. If *a* is the amplitude of SHM, then K.E. is equal to the P.E. at ............ distance from the mean position.
(a) $\frac{a}{\sqrt{2}}$ (b) $\frac{a}{2}$
(c) $\frac{a}{4}$ (d) $a$

34. Energy of a particle in simple harmonic motion depends on
(a) $a^2$ (b) $\omega$
(c) $\frac{1}{a^2}$ (d) $\frac{1}{\omega^2}$

35. The total energy of a particle, executing simple harmonic motion is
(a) independent of x (b) $\propto x^2$
(c) $\propto x$ (d) $\propto x^{1/2}$

36. The potential energy of a particle ($U_x$) executing S.H.M. is given by
(a) $U_x = \frac{k}{2}(x-a)^2$ (b) $U_x = k_1x + k_2x^2 + k_3x^3$
(c) $U_x = Ae^{-bx}$ (d) $U_x$ = a constant

37. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as a function of displacement *x*. Which of the following statement is true?
(a) P.E. is maximum when $x = 0$.
(b) K.E. is maximum when $x = 0$.
(c) T.E. is zero when $x = 0$.
(d) K.E. is maximum when x is maximum.

**38.** If $<E>$ and $<U>$ denote the average kinetic and the average potential energies respectively of mass describing a simple harmonic motion, over one period, then the correct relation is

(a) $<E>=<U>$ (b) $<E>=2<U>$
(c) $<E>=-2<U>$ (d) $<E>=-<U>$

**39.** The total energy of a particle executing S.H.M. is proportional to

(a) Displacement from equilibrium position
(b) Frequency of oscillation
(c) Velocity in equilibrium position
(d) Square of amplitude of motion

**40.** For a particle executing simple harmonic motion, which of the following statement is not correct ?

(a) The total energy of the particle always remains the same.
(b) The restoring force of always directed towards a fixed point.
(c) The restoring force is maximum at the extreme positions.
(d) The acceleration of the particle is maximum at the equilibrium position.

**41.** If a body is executing simple harmonic motion, then

(a) at extreme positions, the total energy is zero
(b) at equilibrium position, the total energy is in the form of potential energy
(c) at equilibrium position, the total energy is in the form of kinetic energy
(d) at extreme positions, the total energy is infinite

**42.** In a simple harmonic oscillator, at the mean position

(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum

**43.** A spring-mass system oscillates with a frequency $\nu$. If it is taken in an elevator slowly accelerating upward, the frequency will

(a) increase (b) decrease
(c) remain same (d) become zero

**44.** Two springs of spring constants $k_1$ and $k_2$ are joined in series. The effective spring constant of the combination is given by

(a) $k_1k_2/(k_1+k_2)$ (b) $k_1k_2$
(c) $(k_1+k_2)/2$ (d) $k_1+k_2$

**45.** Identify the wrong statement from the following

(a) If the length of a spring is halved, the time period of each part becomes $\frac{1}{\sqrt{2}}$ times the original
(b) The effective spring constant K of springs in parallel is given by $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\ldots$
(c) The time period of a stiffer spring is less than that of a soft spring
(d) The spring constant is inversely proportional to the spring length

**46.** The period of oscillation of a simple pendulum swinging through small angles is given by

(a) $T=\pi\sqrt{\frac{L}{g}}$ (b) $T=2\pi\sqrt{\frac{L}{g}}$
(c) $T=\sqrt{\frac{2\pi L}{g}}$ (d) $T=2\pi\sqrt{\frac{g}{L}}$

**47.** For an oscillating simple pendulum, the tension in the string is

(a) maximum at extreme position
(b) maximum at mean position
(c) constant throughout the motion
(d) Cannot be predicted

**48.** The ratio of energies of oscillations of two exactly identical pendulums oscillating with amplitudes 5 cm and 10 cm is :

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

**49.** The period of vibration for a simple pendulum of length $l$ is

(a) $T=2\pi\frac{l}{g}$ (b) $T=2\pi\sqrt{\frac{l}{g}}$
(c) $T=\frac{1}{2}\pi\frac{l}{g}$ (d) $T=\frac{1}{2}\pi\sqrt{\frac{l}{g}}$

**50.** The potential energy of a simple pendulum is maximum when it is

(a) at the turning points of the oscillations
(b) at the equilibrium
(c) in between the above two cases
(d) at any position, it has always a fixed value

**51.** Motion of an oscillating liquid column in a U-tube is

(a) periodic but not simple harmonic
(b) non-periodic
(c) simple harmonic and time period is independent of the density of the liquid.
(d) simple harmonic and time-period is directly proportional to the density of the liquid.

**52.** The necessary condition for the bob of a pendulum to execute SHM is

(a) the length of pendulum should be small
(b) the mass of bob should be small
(c) amplitude of oscillation should be small
(d) the velocity of bob should be small

**53.** The motion which is not simple harmonic is

(a) vertical oscillations of a spring
(b) motion of simple pendulum
(c) motion of a planet around the Sun
(d) oscillation of liquid column in a U-tube

**54.** The tension in the string of a simple pendulum is

(a) constant
(b) maximum in the extreme position
(c) zero in the mean position
(d) None of these

55. The bob of a simple pendulum of mass m and total energy E will have maximum linear momentum equal to
(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$
(c) 2mE (d) $mE^2$

56. How does the time period of pendulum vary with length
(a) $\sqrt{L}$ (b) $\sqrt{\frac{L}{2}}$
(c) $\frac{1}{\sqrt{L}}$ (d) 2L

57. The time period of a simple pendulum of infinite length is ($R_e$ = radius of Earth)
(a) $T = 2\pi\sqrt{\frac{R_e}{g}}$ (b) $T = 2\pi\sqrt{\frac{2R_e}{g}}$
(c) $T = 2\pi\sqrt{\frac{R_e}{2g}}$ (d) $T = \infty$

58. A simple pendulum is set into vibrations. The bob of the pendulum comes to rest after some time due to
(a) Air friction
(b) Moment of inertia
(c) Weight of the bob
(d) Combination of all the above

59. A simple pendulum oscillates in air with time period T and amplitude A. As the time passes
(a) T and A both decrease
(b) T increases and A is constant
(c) T remains same and A decreases
(d) T decreases and A is constant

60. Choose the correct statement
(a) Time period of a simple pendulum depends on amplitude
(b) Time shown by a spring watch varies with acceleration due to gravity g.
(c) In a simple pendulum time period varies linearly with the length of the pendulum
(d) The graph between length of the pendulum and time period is a parabola.

61. Which of the following will change the time period as they are taken to moon?
(a) A simple pendulum (b) A physical pendulum
(c) A torsional pendulum (d) A spring-mass system

62. The total mechanical energy of a harmonic oscillator is given as
(a) $\frac{1}{4}m\omega^2A^2$ (b) $\frac{1}{4}m^2\omega^2A^2$
(c) $\frac{1}{2}m\omega^2A^2$ (d) $\frac{1}{2}m^2\omega^2A^2$

63. In case of sustained forced oscillations the amplitude of oscillations
(a) decreases linearly
(b) decreases sinusoidally
(c) decreases exponentially
(d) always remains constant

64. A particle oscillating under a force $F = \quad k\bar{x} \quad b\bar{\upsilon}$ is a (k and b are constants)
(a) simple harmonic oscillator
(b) non linear oscillator
(c) damped oscillator
(d) forced oscillator

65. The frequency of the simple harmonic motion attained in forced oscillations, after the forced oscillation die out, is
(a) the natural frequency of the particle
(b) the frequency of the driving force
(c) double the frequency of the driving force
(d) double the natural frequency of the particle

66. If a body oscillates at the angular frequency $\omega_d$ of the driving force, then the oscillations are called
(a) free oscillations (b) coupled oscillations
(c) forced oscillations (d) maintained oscillations

67. In case of a forced vibration, the resonance wave becomes very sharp when the
(a) quality factor is small
(b) damping force is small
(c) restoring force is small
(d) applied periodic force is small

68. What is the amplitude of simple harmonic motion at resonance in the ideal case of zero damping?
(a) Zero (b) – 1
(c) 1 (d) Infinite

69. Resonance is an example of
(a) tuning fork (b) forced vibration
(c) free vibration (d) damped vibration

## STATEMENT TYPE QUESTIONS

70. Select the incorrect statement(s) from the following.
I. A simple harmonic motion is necessarily periodic.
II. A simple harmonic motion may be oscillatory
III. An oscillatory motion is necessarily periodic
(a) I only (b) II and III
(c) I and III (d) I and II

71. Select the true statement(s) about SHM.
I. Acceleration ∝ displacement is a sufficient condition for SHM.
II. Restoring force ∝ displacement is a sufficient condition for SHM.
(a) I only (b) II only
(c) I and II (d) None of these

72. Select the false statement(s) from the following.
I. In SHM, the acceleration of the body is in the direction of velocity of the body.
II. In SHM, the velocity and displacement of particle are in same phase.
(a) I only (b) II only
(c) I and II (d) None of these

73. Consider the following statements and select the correct option from the following.
I. PE becomes maximum twice and KE becomes maximum once in one vibration.

II. KE becomes maximum twice and PE become maximum once in one vibration.

III. Both KE and PE becomes maximum twice in one vibration.

(a) I only (b) II only
(c) III only (d) I and II

**74.** Select the correct statement(s) from the following

I. Motion of a satellite and a planet is periodic as well as SHM.

II. Mass suspended by a spring executes SHM.

III. Motion of a simple pendulum is always SHM.

(a) I only (b) II only
(c) I and II (d) I, II and III

**75.** Choose the false statement(s) for a forced oscillation.

I. Displacement amplitude of an oscillator is independent of the angular frequency of the driving force.

II. The amplitude tends to infinity when the driving frequency equals the natural frequency.

III. Maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping.

(a) I only (b) II only
(c) I and II (d) I, II and III

**76.** Select the correct statement(s) from the following.

I. In sitar, guitar the strings vibrate and produce sound.

II. Sound waves propagate due to vibration of air molecules.

III. In solids, atoms oscillate to produce the temperature sensation.

(a) I only (b) II only
(c) I and II (d) I, II and III

**77.** Choose the false statement(s) from the following.

I. Natural frequency of a body depends upon the elastic properties of material of the body.

II. Natural frequency of a body depends on the dimensions of the body.

(a) I only (b) II only
(c) I and II (d) None of these

**78.** Consider the following statements

I. $(\omega t + \phi)$ is known as phase constant.

II. $\phi$ is known as phase constant.

III. $\phi$ is the value of phase at $t = 0$

Choose the correct statement(s)

(a) I only (b) II only
(c) I and II (d) I, II and III

**79.** Consider the following statements

I. Time period of a spring mass system depends on its amplitude.

II. Time period of a spring mass system depends on its mass.

III. Time period of a spring mass system depends on spring constant.

Choose the correct statements.

(a) I and II (b) I and III
(c) II and III (d) I, II and III

## MATCHING TYPE QUESTIONS

**80.** Match the column I and column II.

| | **Column I** | | **Column II** |
|---|---|---|---|
| (A) | Max. positive displacement | (1) | 0 |
| (B) | Max. positive velocity | (2) | $\frac{T}{2}$ |
| (C) | Min. acceleration | (3) | $\frac{T}{4}$ |
| (D) | Max. positive acceleration | (4) | $T$ |
| (E) | Min. displacement | (5) | $\frac{3T}{4}$ |

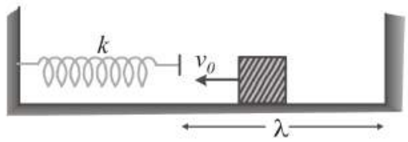
(a) (A) - (2), (B) - (4), (C) - (3), (D) - (5), (E) - (1)
(b) (A) - (1), (B) - (4), (C) - (5), (D) - (3), (E) - (2)
(c) (A) - (5), (B) - (1, 4), (C) - (3), (D) - (5), (E) - (1, 2, 4)
(d) (A) - (1, 3), (B) - (3, 4), (C) - (5), (D) - (1, 2), (E) - (5, 4)

**81.** A particle of mass 2 kg is moving on a straight line under the action of force $F = (8 - 2x)$ N. The particle is released at rest from $x = 6$m. For the subsequent motion match the following (All the values in the column II are in their S.I. units)

| | **Column I** | | **Column II** |
|---|---|---|---|
| (A) | Equilibrium position at $x$ | (1) | $\pi/4$ |
| (B) | Amplitude of SHM is | (2) | $\pi/2$ |
| (C) | Time taken to go directly from $x = 2$ to $x = 4$ | (3) | 4 |
| (D) | Energy of SHM is | (4) | 2 |

(a) (A) - (3), (B) - (4), (C) - (2), (D) - (3)
(b) (A) - (4), (B) - (3), (C) - (2), (D) - (1)
(c) (A) - (1), (B) - (2), (C) - (3), (D) - (4)
(d) (A) - (2), (B) - (4), (C) - (1), (D) - (3)

**82.** A block of mass $m$ is projected towards a spring with velocity $v_0$. The force constant of the spring is $k$. The block is projected from a distance $\ell$ from the free end of the spring. The collision between block and the wall is completely elastic. Match the following columns :




| | **Column-I** | | **Column-II** |
|---|---|---|---|
| (A) | Maximum compression of the spring | (1) | $-\sqrt{\frac{kv_0^2}{m}}$ |
| (B) | Energy of oscillations of block | (2) | $\sqrt{\frac{mv_0^2}{k}}$ |
| (C) | Time period of oscillations | (3) | $\frac{1}{2}mv_0^2$ |
| (D) | Maximum acceleration of the block | (4) | $\left[\frac{2\ell}{v_0} + \pi\sqrt{\frac{m}{k}}\right]$ |

(a) (A) - (2), (B) - (3), (C) - (1), (D) - (4)
(b) (A) - (2), (B) - (3), (C) - (4), (D) - (1)
(c) (A) - (1), (B) - (4), (C) - (3), (D) - (2)
(d) (A) - (1), (B) - (2), (C) - (3), (D) - (4)

**83.** Column I — Column II

| Column I | Column II |
|---|---|
| (A) $\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$ | (1) Resonant vibration |
| (B) $\frac{d^2y}{dt^2} + \omega^2 y = 0$ | (2) Free vibration |
| (C) $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = 0$ | (3) Damped vibration |
| (D) $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega^2 y = F \sin pt$ | (4) Forced vibration |
| | (5) Progressive wave |

(a) (A) - (1), (B) - (3), (C) - (2, 4), (D) - (5)
(b) (A) - (1, 3), (B) - (2, 5), (C) - (3), (D) - (4, 5)
(c) (A) - (5), (B) - (2), (C) - (3), (D) - (1, 4)
(d) (A) - (1), (B) - (2), (C) - (3), (D) - (4)

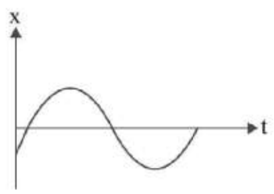
**84.** Column I — Column II

| Column I | Column II |
|---|---|
| (A) Motion of a satellite | (1) Damped oscillations |
| (B) Motion of a simple pendulum | (2) Resonant oscillations |
| (C) Oscillation of stretched string in air | (3) Periodic motion |
| (D) Flying off of a paper rider placed on the stretched string | (4) Simple harmonic motion |

(a) (A) - (2), (B) - (3), (C) - (4), (D) - (1)
(b) (A) - (3), (B) - (2), (C) - (4), (D) - (1)
(c) (A) - (1), (B) - (3), (C) - (2), (D) - (4)
(d) (A) - (3), (B) - (4), (C) - (1), (D) - (2)

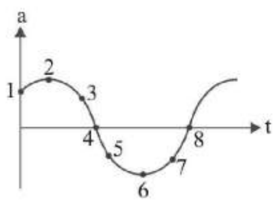
## DIAGRAM TYPE QUESTIONS

**85.** The displacement vs time of a particle executing SHM is shown in figure. The initial phase $\phi$ is



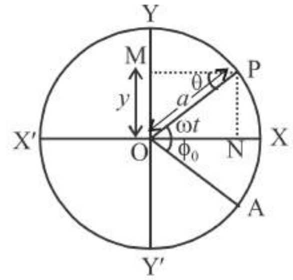

(a) $-\pi < \phi < -\frac{\pi}{2}$
(b) $\pi < \phi < \frac{3\pi}{2}$
(c) $-\frac{3\pi}{2} < \phi < -\pi$
(d) $\frac{\pi}{2} < \phi < \pi$

**86.** The acceleration of a particle undergoing SHM is graphed in figure. At point 2 the velocity of the particle is



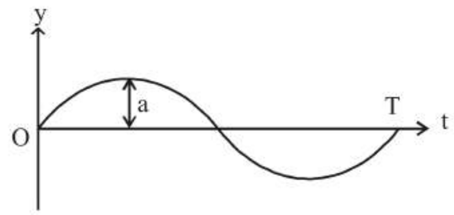

(a) zero
(b) negative
(c) positive
(d) None of these

**87.** For the given figure,



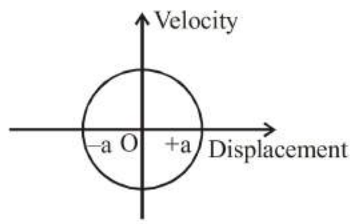

(a) $y = a \sin \omega t$
(b) $y = a \sin (\omega t - \phi_0)$
(c) $y = a \cos \omega t$
(d) $y = a \cos (\omega t - \phi_0)$

**88.** In the given displacement time curve for SHM at what value of $t$ is the amplitude negative?



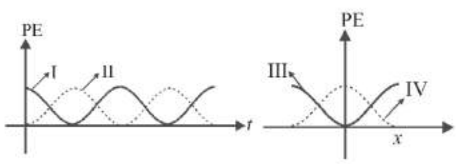

(a) $\frac{T}{2}$
(b) $\frac{T}{4}$
(c) $\frac{3T}{4}$
(d) $\frac{3T}{2}$

**89.** The graph shown in figure represents



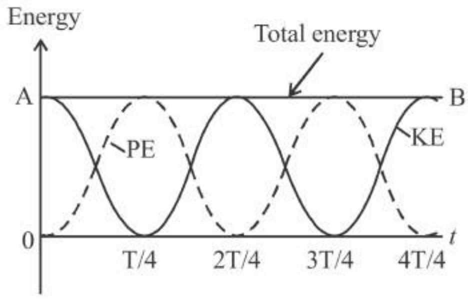

(a) S.H.M.
(b) circular motion
(c) rectillinear motion
(d) uniform circular motion

**90.** For a particle executing SHM the displacement $x$ is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time $t$ and displacement $x$.



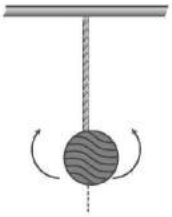

(a) I, III
(b) II, IV
(c) II, III
(d) I, IV

**91.** What do you conclude from the graph about the frequency of KE, PE and SHM ?



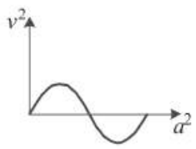

(a) Frequency of KE and PE is double the frequency of SHM
(b) Frequency of KE and PE is four times the frequency SHM.
(c) Frequency of PE is double the frequency of K.E.
(d) Frequency of KE and PE is equal to the frequency of SHM.

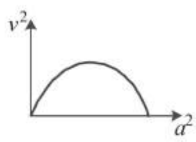
**92.** A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

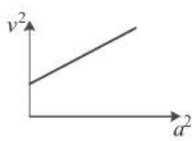


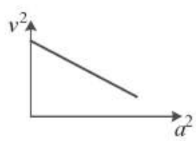
(a) remain unchanged (b) increase
(c) decrease (d) become erratic

**93.** A graph of the square of the velocity against the square of the acceleration of a given simple harmonic motion is

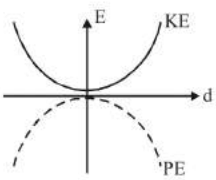
(a) 

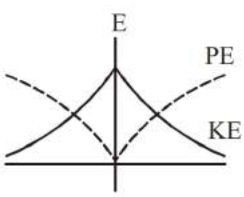

(b) 

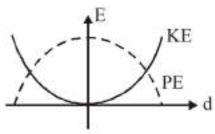

(c) 

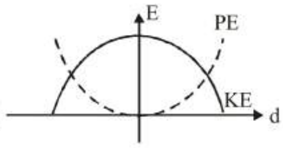

(d) 


**94.** For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*)

(a) 


(b) 


(c) 


(d) 


## ASSERTION- REASON TYPE QUESTIONS

**Directions** : Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

**95.** **Assertion :** An oscillatory motion is necessarily periodic.
**Reason :** A simple harmonic motion is necessarily oscillatory.

**96.** **Assertion :** All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory
**Reason :** Simple pendulum is an example of oscillatory motion.

**97.** **Assertion :** In oscillatory motion, displacement of a body from equilibrium can be represented by *sine* or *cosine* function.
**Reason :** The body oscillates to and fro about its mean position.

**98.** **Assertion :** *Sine* and *cosine* functions are periodic functions.
**Reason :** Sinusoidal functions repeats it values after a difinite interval of time

**99.** **Assertion :** A S.H.M. may be assumed as composition of many S.H.M.'s.
**Reason :** Superposition of many S.H.M.'s (along same line) of same frequency will be a S.H.M.

**100.** **Assertion :** In simple harmonic motion, the motion is to and fro and periodic
**Reason :** Velocity of the particle $(v) = \omega\sqrt{k^2 - x^2}$ (where x is the displacement).

**101.** **Assertion :** In SHM, the the velocity is maximum when acceleration is minimum.
**Reason :** Displacement and velocity of SHM differ in phase by $\frac{\pi}{2}$ rad.

**102.** **Assertion :** The force acting on a particle moving along *x*-axis is
$F = -k(x + v_0 t)$, where $k$ is a constant.
**Reason :** To an observer moving along x-axis with constant velocity $v_0$, it represents SHM.

**103.** **Assertion :** At extreme positions of a particle executing SHM, both velocity and acceleration are zero.
**Reason :** In SHM, acceleration always acts towards mean position.

**104. Assertion :** A particle executing simple harmonic motion comes to rest at the extreme positions .

**Reason :** The resultant force on the particle is zero at these positions.

**105. Assertion :** The graph of total energy of a particle in SHM w.r.t. position is a straight line with zero slope.

**Reason :** Total energy of particle in SHM remains constant throughout its motion.

**106. Assertion :** In a S.H.M. kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude.

**Reason :** In SHM, kinetic energy is zero when potential energy is maximum.

**107. Assertion :** If the amplitude of simple harmonic oscillator is doubled, its total energy becomes four times.

**Reason :** The total energy is directly proportional to the square of amplitude of oscillations.

**108. Assertion :** The graph between velocity and displacement for a harmonic oscillator is an ellipse.

**Reason :** Velocity does not change uniformly with displacement in harmonic motion.

**109. Assertion :** The periodic time of a hard spring is less as compared to that of a soft spring.

**Reason :** The periodic time depends upon the spring constant, and spring constant is large for hard spring.

**110. Assertion:** Time period of a loaded spring does not change when taken to moon.

**Reason:** Time period of a loaded spring depends upon the mass attached and the spring constant and not on acceleration due to gravity *g*.

**111. Assertion:** Pendulum clock will gain time on the mountain top.

**Reason:** On the mountain top the length of the pendulum will decrease and $T \propto \sqrt{l}$, so it will also decrease.

**112. Assertion:** If amplitude of simple pendulum increases then the motion of pendulum is oscillatory but not simple harmonic.

**Reason:** For larger amplitude θ is large and then $\sin\theta \neq \theta$, so the motion is no longer SHM.

**113. Assertion :** When a simple pendulum is made to oscillate on the surface of moon, its time period increases.

**Reason :** Moon is much smaller as compared to earth.

**114. Assertion :** The amplitude of an oscillating pendulum decreases gradually with time

**Reason :** The frequency of the pendulum decreases with time

**115. Assertion :** Damped oscillation indicates loss of energy.

**Reason :** The energy loss in damped oscillation may be due to friction, air resistance etc.

**116. Assertion:** Amplitude of a forced vibration can never reach infinity.

**Reason:** The driving frequency cannot be increased beyond a certain limit.

**117. Assertion :** Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum

**Reason :** The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

## CRITICAL THINKING TYPE QUESTIONS

**118.** A particle executing simple harmonic motion covers a distance equal to half of its amplitude in one second. Then the time period of the simple harmonic motion is

(a) 4 s (b) 6 s (c) 8 s (d) 12 s

**119.** Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the *x*-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

**120.** Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite directions when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. The phase difference is

(a) 0 (b) 2π/3 (c) π (d) π/6

**121.** The displacement of a particle in SHM is $x = 10\sin\left(2t - \frac{\pi}{6}\right)$ metre. When its displacement is 6 m, the velocity of the particle (in m $s^{-1}$) is

(a) 8 (b) 24 (c) 16 (d) 10

**122.** The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

**123.** Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$.

The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{-\pi}{3}$

**124.** A body is executing S.H.M. When its displacement from the mean position are 4 cm and 5 cm, it has velocities 10 cm $s^{-1}$ and 8 cm $s^{-1}$ respectively. Its periodic time is

(a) π/2 s (b) π s (c) 3 π/2 s (d) 2 π s

**125.** A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t$ Ĥ $\pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t$ Ĝ $\delta)$, then
(a) $A = x_0\omega^2, \delta = 3\pi/4$ (b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2, \delta = \pi/4$ (d) $A = x_0\omega^2, \delta = -\pi/4$

**126.** Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?
(a) $\frac{1}{6}$s (b) $\frac{1}{4}$s (c) $\frac{1}{3}$s (d) $\frac{1}{12}$s

**127.** A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is
(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$
(c) $\frac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

**128.** A particle is executing simple harmonic motion with amplitude A. When the ratio of its kinetic energy to the potential energy is $\frac{1}{4}$, its displacement from its mean position is
(a) $\frac{2}{\sqrt{5}}A$ (b) $\frac{\sqrt{3}}{2}A$ (c) $\frac{3}{4}A$ (d) $\frac{1}{4}A$

**129.** A mass of 4 kg suspended from a spring of force constant 800 N $m^{-1}$ executes simple harmonic oscillations. If the total energy of the oscillator is 4 J, the maximum acceleration (in m $s^{-2}$) of the mass is
(a) 5 (b) 15 (c) 45 (d) 20

**130.** The period of oscillation of a mass M suspended from a spring of negligible mass is T. If along with it another mass M is also suspended, the period of oscillation will now be
(a) T (b) $T/\sqrt{2}$
(c) 2T (d) $\sqrt{2}\,T$

**131.** A particle at the end of a spring executes S.H.M with a period $t_1$, while the corresponding period for another spring is $t_2$. If the period of oscillation with the two springs in series is T then
(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

**132.** The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2}$ cos πt metre. The time at which the maximum speed first occurs is
(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s

**133.** When two displacements represented by $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are superimposed the motion is:
(a) simple harmonic with amplitude $\frac{a}{b}$
(b) simple harmonic with amplitude $\sqrt{a^2 + b^2}$
(c) simple harmonic with amplitude $\frac{(a+b)}{2}$
(d) not a simple harmonic

**134.** For a particle moving according to the equation $x = a\cos \pi t$, the displacement in 3 s is
(a) 0 (b) 0.5a
(c) 1.5a (d) 2a

**135.** A particle is executing SHM along a straight line. Its velocities at distances $x_1$ and $x_2$ from the mean position are $V_1$ and $V_2$, respectively. Its time period is
(a) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$ (b) $2\pi\sqrt{\frac{V_1^2 + V_2^2}{x_1^2 + x_2^2}}$
(c) $2\pi\sqrt{\frac{V_1^2 - V_2^2}{x_1^2 - x_2^2}}$ (d) $2\pi\sqrt{\frac{x_1^2 - x_2^2}{V_1^2 - V_2^2}}$

**136.** If the differential equation for a simple harmonic motion is $\frac{d^2y^2}{dt^2} + 2y = 0$, the time-period of the motion is
(a) $\pi\sqrt{2}s$ (b) $\frac{\sqrt{2}s}{\pi}s$
(c) $\frac{\pi}{\sqrt{2}}s$ (d) $2\pi s$

**137.** The total energy of the particle executing simple harmonic motion of amplitude A is 100 J. At a distance of 0.707 A from the mean position, its kinetic energy is
(a) 25 J (b) 50 J
(c) 100 J (d) 12.5 J

**138.** A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then:
(a) amplitude of motion is 3a
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

**139.** A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β. Then its time period of vibration will be :
(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta^2}{\alpha}$
(c) $\frac{2\pi\beta}{\alpha}$ (d) $\frac{\beta^2}{\alpha^2}$

**140.** When the displacement of a particle executing simple harmonic motion is half of its amplitude, the ratio of its kinetic energy to potential energy is `
(a) 1 : 3 (b) 2 : 1
(c) 3 : 1 (d) 1 : 2

**141.** A body oscillates with SHM according to the equation (in SI units), $x = 5\cos\left(2\pi t \frac{\pi}{4}\right)$.

Its instantaneous displacement at $t = 1$ second is

(a) $\frac{\sqrt{2}}{5}$ m (b) $\frac{1}{\sqrt{3}}$ m

(c) $\frac{5}{\sqrt{2}}$ m (d) $\frac{1}{2}$ m

**142.** The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 11% (b) 21% (c) 42% (d) 10%

**143.** The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is $t_0$ in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000\ kg/m^3$. What relationship between t and $t_0$ is true

(a) $t = 2t_0$ (b) $t = t_0/2$

(c) $t = t_0$ (d) $t = 4t_0$

**144.** If a simple pendulum has significant amplitude (upto a factor of $1/e$ of original) only in the period between $t = 0s$ to $t = \tau s$, then $\tau$ may be called the average life of the pendulum. when the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum is (assuming damping the small) in seconds

(a) $\frac{0.693}{b}$ (b) $b$ (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

**145.** Two pendulums of lengths 1.44 m and 1 m start to swing together. The number of vibrations after which they will again start swinging together is

(a) 4 (b) 3 (c) 5 (d) 2

**146.** A simple pendulum has a metal bob, which is negatively charged. If it is allowed to oscillate above a positively charged metallic plate, then its time period will

(a) increase (b) decrease

(c) become zero (d) remain the same

**147.** A block connected to a spring oscillates vertically. A damping force $F_d$, acts on the block by the surrounding medium. Given as $F_d = -bV$ $b$ is a positive constant which depends on :

(a) viscosity of the medium

(b) size of the block

(c) shape of the block

(d) All of these

**148.** The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

**149.** The amplitude of a damped oscillator becomes $\left(\frac{1}{3}\right)^{rd}$ in 2 seconds. If its amplitude after 6 seconds is $\frac{1}{n}$ times the original amplitude, the value of $n$ is

(a) $3^2$ (b) $3^3$ (c) $\sqrt[3]{3}$ (d) $2^3$

**150.** When two springs A and B with force constants $k_A$ and $k_B$ are stretched by the same force, then the respective ratio of the work done on them is

(a) $k_B : k_A$ (b) $k_A : k_B$

(c) $k_A k_B : 1$ (d) $\sqrt{k_B} : \sqrt{k_A}$

**151.** Two oscillating simple pendulums with time periods T and $\frac{5T}{4}$ are in phase at a given time. They are again in phase after an elapse of time

(a) 4T (b) 3T (c) 6T (d) 5T

**152.** A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to $T_M$. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

(g = gravitational acceleration)

(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

**153.** A pendulum of time period 2 s on earth is taken to another planet whose mass and diameter are twice that of earth. Then its time period on the planet is (in second)

(a) $\frac{1}{2}$ (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

**154.** A simple harmonic oscillator of angular frequency 2 rad $s^{-1}$ is acted upon by an external force $F = \sin t$ N. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to

(a) $\sin t + \frac{1}{2}\cos 2t$ (b) $\cos t - \frac{1}{2}\sin 2t$

(c) $\sin t - \frac{1}{2}\sin 2t$ (d) $\sin t + \frac{1}{2}\sin 2t$

**155.** A pendulum with time period of 1s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in $s^{-1}$) is :

(a) $\frac{1}{2}$ (b) $\frac{1}{30}\ln 3$

(c) 2 (d) $\frac{1}{15}\ln 3$

**156.** A cylindrical block of wood (density = 650 kg $m^{-3}$), of base area $30 cm^2$ and height 54 cm, floats in a liquid of density 900 kg $m^{-3}$. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly)

(a) 52 cm (b) 65 cm

(c) 39 cm (d) 26 cm

# HINTS AND SOLUTIONS

## FACT/DEFINITION TYPE QUESTIONS

**1.** **(c)**

**2.** **(a)** The motion of the particle is periodic, (not oscillatory), beacuse it returns to its starting point after a fixed time

**3.** **(a)** $F \propto x$ in SHM.

**4.** **(b)** Since $T$ is independent of mass, so a girl sitting accompanied by another girl will not affect $T$ but if the girl stands up then due to the shift in the location of centre of mass upwards, the effective length decreases and hence $T$ decrease $(T \propto \sqrt{L})$.

**5.** **(a)** $y = 10 \sin (20t + 0.5)$

$y(t) = a \sin (\omega t + \theta)$

$\therefore$ Frequency, $v = \dfrac{\omega}{2\pi} = \dfrac{20}{2\pi}$

**6.** **(c)** General equation of wave motion should be represented by

$y = f(ax \pm bt)$ or $f(at \pm bx)$

when $y$ is a *sine* or *cosine* function such as

$y = a \sin (\omega t - kx)$

or $y = a \cos (\omega t - kx)$

is called simple harmonic progressive wave.

**7.** **(b)** There is no basic difference between oscillation and vibration. For oscillation, frequency is small and for vibration, frequency is high. e.g., oscillation of a pendulum and vibration of a string of a guitar.

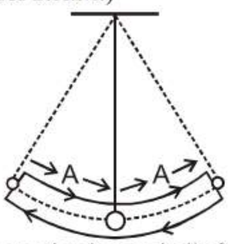
**8.** **(b)**

**9.** **(c)** The motion of particle is circular or elliptic, when two S.H.M. which are perpendicular to each other superimpose on the particle. The particle moves on a

(i) Ellipse if amplitudes of two S.H.M. are different

(ii) Circle, if amplitudes of two S.H.M. are same.

**10.** **(d)** As seen from figure after one time period the bob return to its equilibrium position, so diplacement of the particle is zero, but distance covered by the particle in one time period is 4A (where A is amplitude of bob, when it does S.H.M.)




**11.** **(d)** A periodic motion is a periodic function of time.

$f(t) = A \cos \omega t$ and $f(t) = A \sin \omega t$

A linear combination of the two is

$f(t) = A \sin \omega t + A \cos \omega t$

are all periodic motion equation.

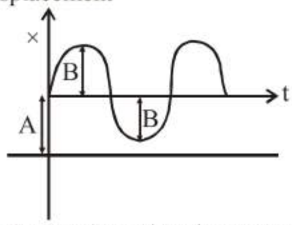
**12.** **(c)** $e^{-2\omega t}$ is not periodic. It decreases with increasing $t$ and never repeats its value.

**13.** **(c)** Clearly $\sin^2 \omega t$ is a periodic function as $\sin \omega t$ is periodic with period $\dfrac{\pi}{\omega}$

**14.** **(b)**

**15.** **(b)** The motion is S.H.M. (as seen from fig.) with an amplitude B.

Displacement



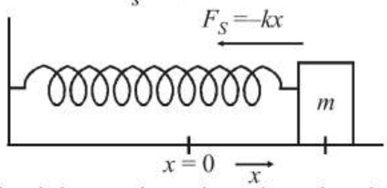

**16.** **(d)** In simple harmonic motion, frequency remains constant and else changes with time.

**17.** **(b)** The simplest type of oscillatory motion is SHM

In SHM

$F \propto x$

$F_s = -kx$




Simple harmonic motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position, which is also the equilibrium position.

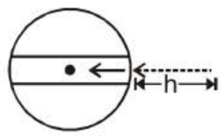
Further, at any point in its oscillation, this force is directed towards the mean position. Displacement and force are in opposite directions.

**18.** **(a)** If $\mathbf{F} \propto \boldsymbol{x}$, this motion is known as linear harmonic motion.

Here, constant proportionality is negative.

If **F** is proportional to higher powers of $x$, this motion is known as non-linear harmonic motion.

**19.** **(c)** When a particle is dropped from a height h above the centre of tunnel.




(i) It will oscillate, through the earth to a height h on both sides.

(ii) The motion of particle is periodic.

(iii) The motion of particle will not be S.H.M.

20. (d) Its motion is complex, but not periodic, oscillatory or S.H.M.

21. (d) The motion of particle is S.H.M. with
$x = A \sin \omega t + B \cos \omega t$
$= a \sin(\omega t + \theta)$
Where $a = \sqrt{A^2 + B^2}$, $A = a \cos\theta$, $B = a \sin\theta$, $\theta = \tan^{-1} B/A$.

22. (b) Elasticity brings the particle towards mean position and inertia needed to cross mean position.

23. (d) $x(t) = A \cos(\omega t + \phi)$

$$V(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a(f) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi)$$

24. (c) Displacement, $y = r \sin \omega t$

$$V = \frac{dy}{dt} = r\omega \cos \omega t$$

$$a = \frac{dV}{dt} = -\omega^2 r \sin \omega t$$

$a = -\omega^2 y$

$\therefore \quad a \propto y$

25. (c) Displacement in simple harmonic motion is given by
$y = a \sin \omega t$ .... (i)
Differentiating with respect to time $t$

$$\frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t)$$

$v = a(\cos \omega t)\omega$

$= a\omega\sqrt{(1 - \sin^2 \omega t)}$

Substituting value of sin $\omega t$ from Eq. (i), we get

$$\therefore \quad v = a\omega\sqrt{\left(1 - \frac{y^2}{a^2}\right)}$$

$$v = a\omega\sqrt{\frac{a^2 - y^2}{a^2}}$$

$$v = \omega\sqrt{a^2 - y^2}$$

26. (d) 27. (b) 28. (d)

29. (d) Let $y = A \sin \omega t$

$$v_{inst} = \frac{dy}{dt} = A\omega \cos \omega t = A\omega \sin(\omega t + \pi/2)$$

Acceleration $= -A\omega^2 \sin \omega t$
$= A\omega^2 \sin(\pi + \omega t)$

$$\therefore \phi = \frac{\pi}{2} = 0.5\pi$$

30. (a) $a = -kX$, $X = x + a$.
In simple harmonic motion acceleration is directly proportional to the displacement from the mean position. Also the acceleration is in the opposite direction of displacement.

31. (d)

32. (d) Total mechanical energy is constant throughout the motion and equals $\frac{1}{2}m\omega^2 A^2$.

33. (a) If displacement of particle is $y$, then

$$KE = \frac{1}{2}m\omega^2(a^2 - y^2)$$

& $$P.E. = \frac{1}{2}m\omega^2 y^2$$

If KE = PE

$$\frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2 - \frac{1}{2}m\omega^2 y^2$$

$$2y^2 = a^2 \quad \therefore y = \frac{a}{\sqrt{2}}$$

34. (a) Energy of a particle in simple harmonic motion is given by

$$E = \frac{1}{2}m\omega^2 a^2 \qquad \therefore \quad E \propto a^2$$

Therefore, the energy in simple harmonic motion depends on $a^2$.

35. (a) At any instant the total energy is

$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA_0^2 = \text{constant}$$

$A_0$ = amplitude
hence U is independent of x.

36. (a) P.E. of body in S.H.M. at an instant,

$$U = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}ky^2$$

If the displacement, $y = (a - x)$ then

$$U = \frac{1}{2}k(a - x)^2 = \frac{1}{2}k(x - a)^2$$

37. (b) $$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\therefore \quad K_{max} = \frac{1}{2}m\omega^2 A^2, \text{ at } x = 0.$$

38. (a) $$E_{av} = U_{av} = \frac{1}{4}m\omega^2 A^2$$

39. (d) $E = \frac{1}{2}m\omega^2 a^2 \Rightarrow E \propto a^2$

40. (d) The acceleration of the particle at equilibrium position is zero.

41. (c)

42. (c) In S.H.M., kinetic energy of particle at any point P is

$$\text{Kinetic energy} = \frac{1}{2}m\omega^2(a^2 - x^2)$$

Potential energy $= \left(\frac{1}{2}m\omega^2 x^2\right)$

Where a is amplitude of particle and x is the distance from mean position.
So, at mean position, $x = 0$

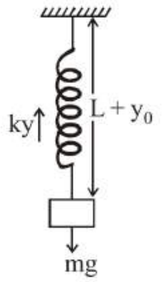
$$K.E. = \frac{1}{2}m\omega^2 a^2 \quad \text{(maximum)}$$

$P.E. = 0$

**43. (c)** For spring mass m system, the time period of the oscillation of mas m is defined as

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{y/g}$$

where m = mass of particle
k = spring constant
y = extension in spring
L = natural length of spring
If g is changed, then y also changes so that y/g is constant, so the time period T of spring mass is independent from the variation in g. Hence ν (frequency) will also not change.




**44. (a)** $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow \frac{1}{k_{eq}} = \frac{k_1 + k_2}{k_1 k_2}$

$$\Rightarrow k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

**45. (b)** In parallel combination, the effective spring constant (K) of springs
$K = K_1 + K_2 + ...$
Hence, option (b) is wrong.

**46. (b)** Time period of a simple pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

**47. (b)** $T = mg\cos\theta$
At the mean position, $\theta = 0°$ and $\cos 0° = 1$
So, the value of tension is greatest.

**48. (c)** $\frac{E_1}{E_2} = \frac{\frac{1}{2}m\omega^2 r_1^2}{\frac{1}{2}m\omega^2 r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{5}{10}\right)^2 = 1:4$

**49. (b)** Time period of a simple pendulum of length $l$ is given by

$T = 2\pi\sqrt{\frac{l}{g}}$ where $g$ is acceleration due to gravity.

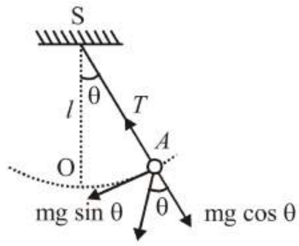
**50. (a)** Potential energy of a simple pendulum is given by

$$U = \frac{1}{2}m\omega^2 y^2$$

Potential energy is maximum when the displacement of the pendulum is maximum, i.e., at the turning points of the oscillations.

**51. (c)**

**52. (c)** Let a simple pendulum be taken as shown. The restoring force on the bob is
$F = -mg\sin\theta$




where $m$ is mass of bob, $g$ the acceleration due to gravity. When angular displacement of bob is small $\sin\theta$ and $\theta$ measured in radians, then

$$\sin\theta \cong \theta = \frac{OA}{SA} = \frac{x}{l}$$

$$\therefore \quad F = -\left(\frac{mg}{l}\right)x \quad \left[\text{angle} = \frac{\text{arc}}{\text{radius}}\right]$$

Since, $m, g$ and $l$ are constant.

$$\therefore \quad F \propto -x$$

Therefore, the motion of bob is simple harmonic.

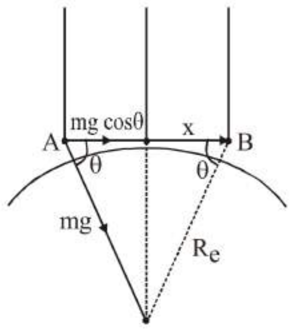
**53. (c)** The motion of a planet around the sun is periodic motion but not a simple harmonic motions.

**54. (d)** Tension is maximum at the mean position.

**55. (b)** $p_{max} = \sqrt{2mE_{max}}$

**56. (a)** Time period $T = 2\pi\sqrt{\frac{L}{g}}$

**57. (a)** The both of the pendulum will more along a straight line AB.



The direction of the Earth's gravitational field is radial

Now, $F = \dfrac{GM_e m}{R_e^2} = mg$

$F_x = -F\cos\theta = -F\dfrac{x}{R_e} = -\dfrac{GM_e m}{R_e^3}x = -kx$

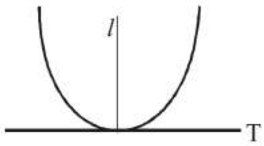
where $k = \dfrac{GM_e m}{R_e{}^3}$

Time period of a simple harmonic oscillator,

$T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m}{GM_e m/R_e^3}}$

or $T = 2\pi\sqrt{\dfrac{R_e}{\dfrac{GM_e}{R_e^2}}} = 2\pi\sqrt{\dfrac{R_e}{g}}$

**58.** **(a)** **59.** **(c)**




**60.** **(d)** $T = 2\pi\sqrt{\dfrac{l}{g}}$

$l = \left(\dfrac{g}{4\pi^2}\right)T^2$

$\therefore\ l \propto T^2$

**61.** **(a)**

**62.** **(c)** Kinetic energy, $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$

Potential energy, $U = \dfrac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$

$\therefore$ Total mech. energy $= \dfrac{1}{2}m\omega^2 A^2$

**63.** **(a)** In case of sustained force oscillations the amplitude of oscillations decreases linearly.

**64.** **(c)** A particle oscillating under a force $\overline{F} = -k\overline{x} - b\overline{\upsilon}$ is a damped oscillator. the first term $-k\overline{x}$ represents the restoring force and second term $-b\overline{\upsilon}$ represents the damping force.

**65.** **(b)**

**66.** **(c)** In forced oscillations, the body oscillates at the angular frequency of the driving force.

**67.** **(b)** The resonance wave becomes very sharp when damping force is small.

**68.** **(d)** **69.** **(b)**

## STATEMENT TYPE QUESTIONS

**70.** **(b)** Every simple harmonic motion (S.H.M.) is necessarily periodic, but a periodic motion may or may not be simple harmonic motion.

**71.** **(b)**

**72.** **(c)** In SHM, acceleration of particle is always directed towards mean position but velocity is either towards or away from mean position. Similarly displacement is always away from the position. So they can not be in phase always.

**73.** **(c)** In one vibration the particle goes twice to extreme positions and twice crosses the mean position. So does the PE and KE.

**74.** **(b)**

**75.** **(a)** Displacement amplitude of an oscillator depends on the angular frequency of the driving force.

**76.** **(d)** Examples of oscillatory motion can be found around us very easily.

In most of the musical instruments either string or some memberane oscillates to produce pleasant sound. Due to the oscillation of air molecules sound propagates in a air.

**77.** **(d)**

**78.** **(b)** $(\omega t + \phi)$ represents the phase of the particle in SHM. $\phi$ represents the phase constant and is the value of phase at $t = 0$

**79.** **(c)** $F_s = -K\,x$ (spring force) ...(i)

$F = -m\omega^2 x$ (For SHM condition) ...(ii)

On comparing Eqs. (i) and (ii),

$\omega = \sqrt{\dfrac{k}{m}} \Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}}$ $\left(\because T = \dfrac{2\pi}{\omega}\right)$

$\Rightarrow T \propto \sqrt{m} \Rightarrow T \propto \dfrac{1}{\sqrt{k}}$

So, $T$ does not depends on the amplitude of the oscillation $T$ depend on m, k.

## MATCHING TYPE QUESTIONS

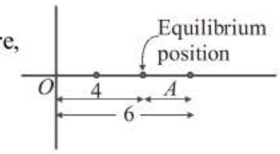
**80.** **(c)**

**81.** **(a)**

(A) For equilibrium, $F = 8 - 2x = 0$

or $x = 4m$

(B) From figure,




$A = 2m$

(C) Time taken from $x = 2$ to $4$

or A to O is $\dfrac{T}{4}$, which differs in phase by $\dfrac{\pi}{2}$.

(D) Energy of SHM, $E = \int_4^6 F dx$

$= \int_4^6 (8 - 2x)dx = |8x - x^2|_4^6$

$= 4$ J.

82. (b) $\frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2$ or $x_0 = \sqrt{\frac{mv_0^2}{k}}$

Time period, $T' = \left[2\frac{T}{4} + 2t\right]$

$= \left[\frac{2\pi}{2}\sqrt{\frac{m}{k}} + 2\frac{\ell_0}{v_0}\right]$

Energy of oscillation $= \frac{1}{2}mv_0^2$.

83. (c) 84. (d)

## DIAGRAM TYPE QUESTIONS

85. (a) For $x = (-A)$, we have

$-A = A\sin(\omega \times 0 + \phi_0)$

or $\phi_0 = -\frac{\pi}{2}$.

So for $x < (-A)$, $\phi_0 < (-\pi/2)$.

86. (a) At point 2, the acceleration of the particle is maximum, which is at the extreme position. At extreme position, the velocity of the particle will be zero.

87. (b) $\frac{y}{a} = \sin\theta$

$\therefore \ y = a\sin\theta$

$\theta = \angle XOP = \omega t - \phi_0$

$\therefore \ y = a\sin(\omega t - \phi_0)$

88. (c)

89. (a) t = 0, v maximum. The motion begins from mean position. So it represents S.H.M.

90. (a) In $x = A\cos\omega t$, the particle starts oscillating from extreme position. So at $t$ = 0, its potential energy is maximum.

91. (a) KE and PE completes two vibration in a time during which SHM completes one vibration. Thus frequency of PE or KE is double than that of SHM.

92. (b) When some mercury is drained off, the centre of gravity of the bob moves down and so length of the pendulum increases, which result increase in time period.

93. (d) $v^2 = \omega^2(A^2 - x^2)$ ...(i)

and $a^2 = (\omega^2 x)^2 = \omega^4 x^2$ ...(ii)

From above equations, we have

$v^2 = -\frac{a^2}{\omega^2} + \omega^2 A^2 \Rightarrow y = mx + c$

It represents straight line with negative slope.

94. (d) $K.E = \frac{1}{2}k(A^2 - d^2)$

and $P.E. = \frac{1}{2}kd^2$

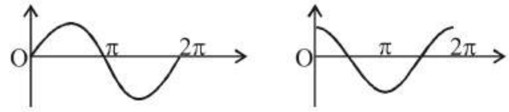
At mean position $d = 0$. At extrement positions $d = A$

## ASSERTION- REASON TYPE QUESTIONS

95. (d) Damped oscillations are non-periodic.

96. (b) 97. (b)

98. (a) A periodic function is one whose value repeats after a definite interval of time sin θ and cos θ are periodic functions because they repeat itself after 2π interval of time




sin curve cos curve

99. (b) So rod remains stationary after the release.

100. (c) S.H.M. is to and fro motion of an object and it is periodic.

$v = \omega\sqrt{k^2 - x^2}$

If x = 0, v has maximum value. At x = k, v has minimum velocity. Similarly, when x = – k, v has zero value, all these indicate to and from movement.

101. (b) $x = A\sin\omega t$

and $v = \omega A\cos\omega t = \omega A\sin(\omega t + \pi/2)$

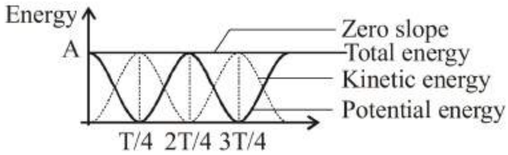
102. (a) With respect to an observer, the force on the particle $F = -k[x + (v_0 - v_0)t] = -kx$, so it represents SHM.

103. (d) At extreme position, $a = \omega^2 A$ and $v = 0$.

104. (c) The force at the extreme position is, $F = m\omega^2 A$.

105. (a) The total energy of S.H.M = Kinetic energy of particle + potential energy of particle

The variation of total energy of the particle in SHM with time is shown in a graph




106. (b) In SHM. $K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$ and $P.E. = \frac{1}{2}m\omega^2 y^2$.

For K.E. = P.E. $\Rightarrow 2y^2 = a^2 \Rightarrow y = a/\sqrt{2}$. Since total energy remains constant through out the motion, which is E = K.E. + P.E. So, when P.E. is maximum then K.E. is zero and viceversa.

107. (a) As $E \propto A^2$, $E' = 4E$.

108. (c)

109. (a) The time period of a oscillating spring is given by

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$, Since the spring constant is large for hard spring, therefore hard spring has a less periodic time as compared to soft spring.

110. (a)

**111. (d)** Both assertion and reason are wrong. At the mountain top $g$ will decrease and $T \propto \frac{1}{\sqrt{g}}$.

It will increase. Thus the pendulum clock will become slow. So, pendulum clock loses time.

**112. (a)**

**113. (b)** At moon $T = 2\pi\sqrt{\frac{\ell}{(g/6)}}$, so time period increases.

**114. (c)** The amplitude of an oscillating pendulum decreases with time because of friction due to air. Frequency of pendulum is independent $\left(T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)$ of amplitude.

**115. (b)** Energy of damped oscillator at an any instant t is given by

$E = E_0 e^{-bt/m}$ [where $E_0 = \frac{1}{2}kx^2$ = maximum energy]

Due to damping forces the amplitude of oscillator will go on decreasing with time whose energy is expressed by above equation.

**116. (c)** Damping can never be zero in reality, so amplitude can never be infinity.

**117. (c)** Amplitude of oscillation for a forced damped oscillatory is $A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2) + (b\omega / m)^2}}$, where b is constant related to the strength of the resistive force, $\omega_0 = \sqrt{k/m}$ is natural frequency of undamped oscillator $(b = 0)$

When the frequency of driving force $(\omega) \approx \omega_0$, then amplitude A is very larger.

For $\omega < \omega_0$ or $\omega > \omega_0$, the amplitude decreases.

## CRITICALTHINKING TYPE QUESTIONS

**118. (d)** According to equation of SHM

$x = A \sin\omega t$

Here, $x = \frac{A}{2}$ and $t = 1$

$\therefore \frac{A}{2} = A \sin \omega \times 1$

$= A \sin \omega$

or, $\sin \omega = \frac{1}{2}, \omega = \frac{\pi}{6}$

$\therefore \frac{2\pi}{T} = \frac{\pi}{6}$ or $T = 12$ s

**119. (a)** Let $x_1 = A\sin(\omega t + \phi_1)$ and $x_2 = A\sin(\omega t + \phi_2)$

$x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t - \phi_1)]$

$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{1}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$

The resultant motion can be treated as a simple harmonic motion with amplitude $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$.

Given, maximum distance between the particles

$\therefore$ Amplitude of resultant S.H.M.

$= X_0 + A - X_0 = A$

$\therefore \quad 2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A.$

$\Rightarrow \quad \phi_2 - \phi_1 = \pi/3.$

**120. (b)** Equation of S.H.M. is given by

$x = A\sin(\omega t + \delta)$

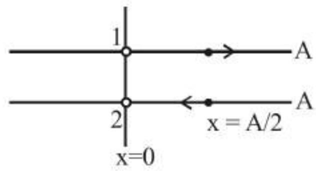
$(\omega t + \delta)$ is called phase.

When $x = \frac{A}{2}$, then

$\sin(\omega t + \delta) = \frac{1}{2}$

$\Rightarrow \omega t + \delta = \frac{\pi}{6}$

or $\phi_1 = \frac{\pi}{6}$




For second particle,

$\phi_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore \phi = \phi_2 - \phi_1 = \frac{4\pi}{6} = \frac{2\pi}{3}$

**121. (c)** Given, $x = 10\sin\left(2t - \frac{\pi}{6}\right)$

$A = 10$ and $\omega = 2$ Hz

$\therefore \upsilon = \omega\sqrt{A^2 - d^2} = 2\sqrt{(10)^2 - (6)^2}$

$= 2\sqrt{100 - 36} = 2 \times 8 = 16\text{m s}^{-1}$

**122. (a)** Maximum velocity,

$v_{max} = a\omega$

$v_{max} = a \times \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01$ s

**123. (b)** $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{dy_2}{dt} = -0.1\pi\sin\pi t = 0.1\pi\cos\left(\pi t + \frac{\pi}{2}\right)$

$\therefore$ Phase diff. $= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$

**124. (b)** Using $v^2 = \omega^2(a^2 - y^2)$ we have

$10^2 = \omega^2(a^2 - 4^2)$ and $8^2 = \omega^2(a^2 - 5^2)$;

so $10^2 - 8^2 = \omega^2(5^2 - 4^2) = (3\omega^2)$ or $6 = 3\omega$ or $\omega = 2$

or $T = 2\pi/\omega = 2\pi/2 = \pi$ s.

**125. (a)** Here,

$x = x_0 \cos(\omega t - \pi/4)$

$\therefore$ Velocity,

$$v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration,

$$a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0\omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= x_0\,\omega^2$$

$$\cos\left(\omega t + \frac{3\pi}{4}\right) \quad ...(1)$$

Acceleration, $a = A\cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$

**126. (a)** K.E. of a body undergoing S.H.M. is given by

$$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t$$

$$T.E. = \frac{1}{2}ma^2\omega^2$$

Given K.E. = 0.75 T.E.

$$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{\pi}{6\times\omega} \Rightarrow t = \frac{\pi\times 2}{6\times 2\pi} \Rightarrow t = \frac{1}{6}\text{ s}$$

**127. (b)** The kinetic energy of a particle executing S.H.M. is given by

$$K = \frac{1}{2}ma^2\omega^2\sin^2\omega t$$

where, m = mass of particle

a = amplitude

$\omega$ = angular frequency

t = time

Now, average K.E. = $\langle K \rangle$

$$= \left\langle \frac{1}{2}m\omega^2a^2\sin^2\omega t \right\rangle$$

$$= \frac{1}{2}m\omega^2a^2\langle\sin^2\omega t\rangle$$

$$= \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right) \quad \left(\because \langle\sin^2\theta\rangle = \frac{1}{2}\right)$$

$$= \frac{1}{4}m\omega^2a^2$$

$$= \frac{1}{4}ma^2(2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$$

or, $\langle K \rangle = \pi^2ma^2\nu^2$

**128. (a)** As we know,

kinetic energy $= \frac{1}{2}m\omega^2(A^2 - x^2)$

Potential energy $= \frac{1}{2}m\omega^2x^2$

$$\therefore \frac{\frac{1}{2}m\omega^2(A^2-x^2)}{\frac{1}{2}m\omega^2x^2} = \frac{1}{4} \Rightarrow \frac{A^2-x^2}{x^2} = \frac{1}{4}$$

$$4A^2 - 4x^2 = x^2 \qquad \Rightarrow x^2 = \frac{4}{5}A^2$$

$$x = \frac{2}{\sqrt{5}}A.$$

**129. (d)** Here, $m = 4$ kg; $k = 800\text{ Nm}^{-1}$; $E = 4$ J

In SHM, $E = \frac{1}{2}kA^2$

$$\therefore 4 = \frac{1}{2}\times 800\times A^2$$

$$A^2 = \frac{8}{800} = \frac{1}{100}, A = 0.1\text{ m}$$

Maximum acceleration, $a_{max} = \omega^2A$

$$= \frac{k}{m}A \quad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$$

$$= \frac{800\text{ Nm}^{-1}}{4\text{ kg}}\times 0.1\text{ m} = 20\text{ ms}^{-2}$$

**130. (d)** $T = 2\pi\sqrt{\frac{m}{K}} \qquad \therefore \frac{T_1}{T_2} = \sqrt{\frac{M_1}{M_2}}$

$$\therefore T_2 = T_1\sqrt{\frac{M_2}{M_1}} = T_1\sqrt{\frac{2M}{M}}$$

$$T_2 = T_1\sqrt{2} = \sqrt{2}\,T \text{ (where } T_1 = T)$$

**131. (b)** $t_1 = 2\pi\sqrt{\frac{m}{k_1}}, t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

when springs are in series then, $k_{eff} = \frac{k_1k_2}{k_1+k_2}$

$$\therefore T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k_2}+\frac{m}{k_1}} = 2\pi\sqrt{\frac{t_2^2}{(2\pi)^2}+\frac{t_1^2}{(2\pi)^2}}$$

$$\Rightarrow T^2 = t_1^2 + t_2^2$$

**132. (b)** Here, $x = 2 \times 10^{-2} \cos \pi t$

Speed is given by

$$v = \frac{dx}{dt} = -2 \times 10^{-2} \pi \sin \pi t$$

For the first time, the speed to be maximum,

$\sin \pi t = 1$

or, $\sin \pi t = \sin \frac{\pi}{2}$

$\Rightarrow \quad \pi t = \frac{\pi}{2}$ or $t = \frac{1}{2} = 0.5$ sec.

**133. (b)** The two displacements equations

are $y_1 = a \sin(\omega t)$

and $y_2 = b \cos(\omega t) = b \sin\left(\omega t + \frac{\pi}{2}\right)$

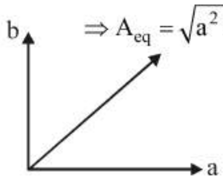
$$y_{eq} = y_1 + y_2$$

$$= a \sin\omega t + b \cos\omega t = a \sin\omega t + b \sin\left(\omega t + \frac{\pi}{2}\right)$$

Since the frequencies for both SHMs are same, resultant motion will be SHM.

Now $A_{eq} = \sqrt{a^2 + b^2 + 2ab \cos \frac{\pi}{2}}$

$\Rightarrow A_{eq} = \sqrt{a^2 + b^2}$




**134. (d)** Initially $t = 0$

$x = a \cos\pi t = - a \cos 0° = a$

Finally at $t = 3$

$x = a \cos 3\pi = - 3a$

Total displacement = 2a

**135. (a)** As we know, for particle undergoing SHM,

$$V = \omega\sqrt{A^2 - X^2}$$

$$V_1^2 = \omega^2 (A^2 - x_1^2)$$

$$V_2^2 = \omega^2 (A^2 - x_2^2)$$

Substructing we get,

$$\frac{V_1^2}{\omega^2} + x_1^2 = \frac{V_2^2}{\omega^2} + x_2^2$$

$$\Rightarrow \frac{V_1^2 - V_2^2}{\omega^2} = x_2^2 - x_1^2$$

$$\Rightarrow w = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$$

**136. (a)** The differential equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} + 2y = 0 \text{ or } \frac{d^2y}{dt^2} = -2y \quad \text{...(i)}$$

Standard equation of simple harmonic motion is

$$\frac{d^2y}{dt^2} = -\omega^2 y \quad \text{...(ii)}$$

Comparing eq. (i) and (ii),

$\omega^2 = 2$ or $\omega = \sqrt{2}$

As we know, $\omega = \frac{2\pi}{T}$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \pi\sqrt{2}s$

**137. (b)** In SHM, Total energy, $E_{total} = \frac{1}{2} m\omega^2 A^2$

and, Kinetic energy, $E_K = \frac{1}{2} m\omega^2 (A^2 - x^2)$

where $x$ is the distance from the mean position.

At $x = 0.707A$

$$E_K = \frac{1}{2} m\omega^2 (A^2 - (0.707A^2) = \frac{1}{2} m\omega^2 (0.5A^2)$$

As per question, $E_{total} = 100$ J

$$\therefore \ E_K = 0.5\left(\frac{1}{2} m\omega^2 A^2\right) = 0.5 \times 100J = 50J$$

**138. (d)** In simple harmonic motion, starting from rest,

At $t = 0$, $x = A$

$x = A\cos\omega t$ ..... (i)

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

From equation (i)

$A - a = A\cos\omega\tau$ ......(ii)

$A - 3a = A \cos 2\omega\tau$ ......(iii)

As $\cos 2\omega\tau = 2\cos^2 \omega\tau - 1$ ...(iv)

From equation (ii), (iii) and (iv)

$$\frac{A - 3a}{A} = 2\left(\frac{A - a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A - 3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$\Rightarrow 2a^2 = aA$

$\Rightarrow A = 2a$

$\Rightarrow \frac{a}{A} = \frac{1}{2}$

Now, $A - a = A \cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{A - a}{A}$$

$\Rightarrow \quad \cos\omega\tau = \frac{1}{2} \quad \text{or} \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$

$\Rightarrow \quad T = 6\tau$

**139. (c)** As, we know, in SHM
Maximum acceleration of the particle, $\alpha = A\omega^2$
Maximum velocity, $\beta = A\omega$

$\Rightarrow \omega = \frac{\alpha}{\beta}$

$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi\beta}{\alpha} \quad \left[\because \omega = \frac{2\pi}{T}\right]$

**140. (c)** Given, $x = \frac{A}{2}$

$\therefore$ from $x = A \sin \omega t$

$\Rightarrow \omega t = 30°$

And, $\frac{KE}{PE} = \cot^2 \omega t = \left(\sqrt{3}\right)^2$

$= 3$

**141. (c)** Given $t = 1$s

$\therefore x = 5\cos\left(2\pi + \frac{\pi}{4}\right)$

$= 5\cos\frac{\pi}{4} = \frac{5}{\sqrt{2}}$ m

i.e., displacement at $t = 1$s is $\frac{5}{\sqrt{2}}$ m

**142. (d)** $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\log T = \log(2\pi) + \frac{1}{2}\log\left(\frac{\ell}{g}\right)$

$\Rightarrow \quad \log T = \log(2\pi) + \frac{1}{2}\log(\ell) - \frac{1}{2}\log(g)$

Differentiating, $\frac{\Delta T}{T} = 0 + \frac{1}{2} \times \frac{\Delta\ell}{\ell} - 0$

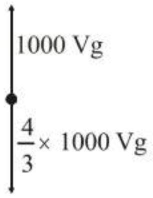
$\Rightarrow \quad \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times \frac{\Delta\ell}{\ell} \times 100$

$= \frac{1}{2} \times 21 = 10.5 \approx 10\%$

***Note:*** In this method, the % error obtained is an approximate value on the higher side. Exact value is less than the obtained one.

**143. (a)** $t = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$ ; $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$

Bob in air




Bob in water

Net force $= \left(\frac{4}{3} - 1\right) \times 1000 \text{ Vg} = \frac{1000}{3}\text{Vg}$

$g_{eff} = \frac{1000Vg}{3 \times \frac{4}{3} \times 1000V} = \frac{g}{4} \left[\text{Mass} = \frac{4}{3} \times 1000 \times V\right]$

$\therefore t = 2\pi\sqrt{\frac{\ell}{g/4}}$

$\therefore \quad t = 2t_0$

**144. (d)** For damped harmonic motion,

$ma = -kx - mbv$

or $\quad ma + mbv + kx = 0$

Solution to above equation is

$x = A_0 e^{-\frac{bt}{2}} \sin\omega t;$ with $\omega^2 = \frac{k}{m} - \frac{b^2}{4}$

where amplitude drops exponentially with time

i.e., $\quad A_\tau = A_0 e^{-\frac{b\tau}{2}}$

Average time $\tau$ is that duration when amplitude drops by 63%, i.e., becomes $A_0/e$.

Thus, $\quad A_\tau = \frac{A_0}{e} = A_0 e^{-\frac{b\tau}{2}}$

or $\quad \frac{b\tau}{2} = 1$ or $\tau = \frac{2}{b}$

**145. (c)** $(n+1)T_S = nT_L$

$(n+1)\sqrt{1} = n\sqrt{1.44}$

$\Rightarrow (n+1) = 1.2n \quad \Rightarrow n = \frac{1}{0.2} = 5$

**146. (b)**

**147. (d)** $F = -bV$, $b$ depends on all the three i.e, shape and size of he block and viscosity of the medium.

**148. (c)** $\because A = A_0 e^{-\frac{bt}{2m}}$ (where, $A_0$ = maximum amplitude)

According to the questions, after 5 seconds,

$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \ldots(i)$

After 10 more seconds,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \ldots(ii)$$

From equations (i) and (ii)

$A = 0.729 A_0$

$\therefore \alpha = 0.729$

**149. (b)** Amplitude of a damped oscillator at any instant $t$ is given by

$A = A_0 e^{-bt/2m}$

where $A_0$ is the original amplitude

From question,

When $t = 2$ s, $A = \frac{A_0}{3}$

$\therefore \frac{A_0}{3} = A_0 e^{-2b/2m}$

or, $\frac{1}{3} = e^{-b/m} \quad \ldots(i)$

When $t = 6$ s, $A = \frac{A_0}{n}$

$\therefore \frac{A_0}{n} = A_0 e^{-6b/2m}$

or, $\frac{1}{n} = e^{-3b/m} = (e^{-b/m})^3$

or, $\frac{1}{n} = \left(\frac{1}{3}\right)^3$ (Using eq. (i))

$\therefore n = 3^3$

**150. (a)** Force on a spring $F = k.x$

$$W = \frac{1}{2}kx^2 = \frac{F.x}{2} = \frac{F}{2}.\frac{F}{k} = \frac{F^2}{2k}$$

Now, $\frac{W_1}{W_2} = \frac{k_2}{k_1} = \frac{k_B}{k_A}$

**151. (d)**

$$(n+1)T = \frac{5T}{4} \times n$$

$$n = 4$$

$$\text{Time } t = (n+1)\,T = 5T$$

**152. (c)** As we know, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell + \Delta\ell}{\ell}} \text{ or } \left(\frac{T_M}{T}\right)^2 = \frac{\ell + \Delta\ell}{\ell}$$

or, $\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{Ay} \quad \left[\because \Delta\ell = \frac{Mg\ell}{Ay}\right]$

$$\therefore \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

**153. (b)** The time period of pendulam is given by

$$T = 2\pi\frac{l}{g}$$

Acceleration due to gravity of earth is

$$g_e = \frac{GM}{R_e^2}$$

Value of 'g' on planet is

$$g_p = \frac{GM_p}{R_p^2} = \frac{G.2M}{4R^2} = \frac{g_e}{2}$$

$$\therefore T_p = 2\pi\sqrt{\frac{l.2}{g_e}} = \sqrt{2}T$$

i.e. $T_p = 2\sqrt{2}$

**154. (c)** As we know,

$F = ma \Rightarrow a \propto F$

or, $a \propto \sin t$

$$\Rightarrow \frac{dv}{dt} \propto \sin t$$

$$\Rightarrow \int_0^0 dV \propto \int_0^t \sin t \, dt$$

$V \propto -\cos t + 1$

$$\int_0^x dx = \int_0^t (-\cos t + 1)\, dt$$

$$x = \sin t - \frac{1}{2}\sin 2t$$

**155. (d)** As we know, $E = E_0 e^{-\frac{bt}{m}}$

$$15 = 45e^{-\frac{b15}{m}}$$

[As no. of oscillations = 15 so $t$ = 15sec]

$$\frac{1}{3} = e^{-\frac{b15}{m}}$$

Taking log on both sides

$$\frac{b}{m} = \frac{1}{15}\ell n 3$$

**156. (c)** $h$ = Length of block immerged in water

$mg = F_B$

$\rho A l g = \rho_B A h g$

$650 \times A \times 54 \times 10^{-2} g = 900 \times A \times hg$

$\Rightarrow h = 0.39\ m = 39$ cm.